4.3 Prove Triangles Congruent by SSS

Before You used the definition of congruent figures.

Now You will use the side lengths to prove triangles are congruent.

Why So you can determine if triangles in a tile floor are congruent, as in Ex. 22.

Key Vocabulary

- **congruent figures,** *p. 225*
- **corresponding parts,** *p. 225*

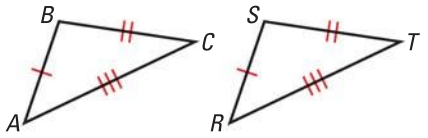
In the Activity on page 233, you saw that there is only one way to form a triangle given three side lengths. In general, any two triangles with the same three side lengths must be congruent.

POSTULATE *For Your Notebook*

POSTULATE 19 Side-Side-Side (SSS) Congruence Postulate

If three sides of one triangle are congruent to three sides of a second triangle, then the two triangles are congruent.

If Side $\overline{AB} \cong \overline{RS}$,
Side $\overline{BC} \cong \overline{ST}$, and
Side $\overline{CA} \cong \overline{TR}$,
then $\triangle ABC \cong \triangle RST$.

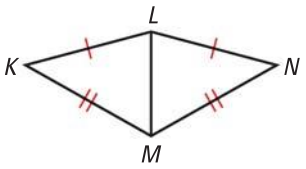


EXAMPLE 1 Use the SSS Congruence Postulate

Write a proof.

GIVEN ▶ $\overline{KL} \cong \overline{NL}$, $\overline{KM} \cong \overline{NM}$

PROVE ▶ $\triangle KLM \cong \triangle NLM$



Proof It is given that $\overline{KL} \cong \overline{NL}$ and $\overline{KM} \cong \overline{NM}$. By the Reflexive Property, $\overline{LM} \cong \overline{LM}$. So, by the SSS Congruence Postulate, $\triangle KLM \cong \triangle NLM$.

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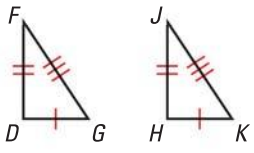
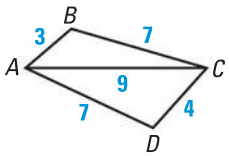
✓ GUIDED PRACTICE for Example 1

Decide whether the congruence statement is true. *Explain* your reasoning.

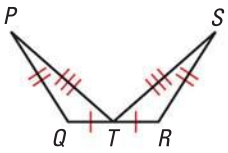
1. $\triangle DFG \cong \triangle HJK$



2. $\triangle ACB \cong \triangle CAD$

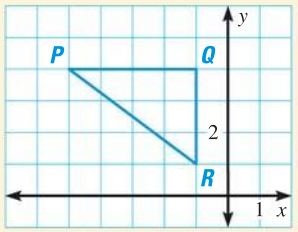


3. $\triangle QPT \cong \triangle RST$



EXAMPLE 2 Standardized Test Practice

Which are the coordinates of the vertices of a triangle congruent to $\triangle PQR$?



(A) $(-1, 1), (-1, 5), (-4, 5)$

(B) $(-2, 4), (-7, 4), (-4, 6)$

(C) $(-3, 2), (-1, 3), (-3, 1)$

(D) $(-7, 7), (-7, 9), (-3, 7)$

ELIMINATE CHOICES

Once you know the side lengths of $\triangle PQR$, look for pairs of coordinates with the same x-coordinates or the same y-coordinates. In Choice C, $(-3, 2)$ and $(-3, 1)$ are only 1 unit apart. You can eliminate D in the same way.

Solution

By counting, $PQ = 4$ and $QR = 3$. Use the Distance Formula to find PR.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$PR = \sqrt{(-1 - (-5))^2 + (1 - 4)^2} = \sqrt{4^2 + (-3)^2} = \sqrt{25} = 5$$

By the SSS Congruence Postulate, any triangle with side lengths 3, 4, and 5 will be congruent to $\triangle PQR$. The distance from $(-1, 1)$ to $(-1, 5)$ is 4. The distance from $(-1, 5)$ to $(-4, 5)$ is 3. The distance from $(-1, 1)$ to $(-4, 5)$ is $\sqrt{(5 - 1)^2 + ((-4) - (-1))^2} = \sqrt{4^2 + (-3)^2} = \sqrt{25} = 5$.

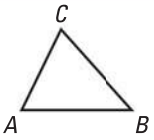
▶ The correct answer is A. (A) (B) (C) (D)

GUIDED PRACTICE for Example 2

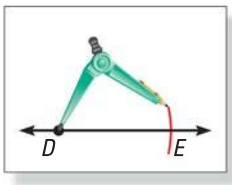
4. $\triangle JKL$ has vertices $J(-3, -2)$, $K(0, -2)$, and $L(-3, -8)$. $\triangle RST$ has vertices $R(10, 0)$, $S(10, -3)$, and $T(4, 0)$. Graph the triangles in the same coordinate plane and show that they are congruent.

ACTIVITY COPY A TRIANGLE

Follow the steps below to construct a triangle that is congruent to $\triangle ABC$.

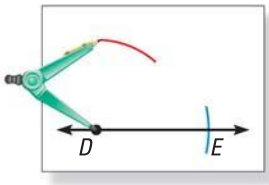


STEP 1



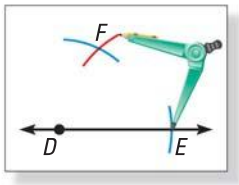
Construct $\overline{DE}$ so that it is congruent to $\overline{AB}$.

STEP 2



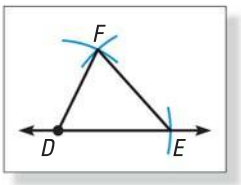
Open your compass to the length AC. Use this length to draw an arc with the compass point at D.

STEP 3



Draw an arc with radius BC and center E that intersects the arc from Step 2. Label the intersection point F.

STEP 4



Draw $\triangle DEF$. By the SSS Congruence Postulate, $\triangle ABC \cong \triangle DEF$.

EXAMPLE 3 Solve a real-world problem

STRUCTURAL SUPPORT Explain why the bench with the diagonal support is stable, while the one without the support can collapse.

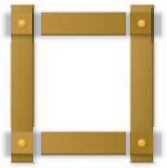
Solution

The bench with a diagonal support forms triangles with fixed side lengths. By the SSS Congruence Postulate, these triangles cannot change shape, so the bench is stable. The bench without a diagonal support is not stable because there are many possible quadrilaterals with the given side lengths.

GUIDED PRACTICE for Example 3

Determine whether the figure is stable. *Explain* your reasoning.

5.



6.

7.

4.3 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7, 9, and 25

★ = **STANDARDIZED TEST PRACTICE** Exs. 16, 17, and 28

SKILL PRACTICE

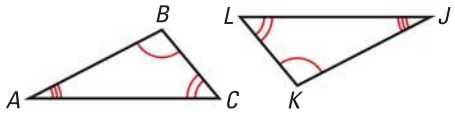
VOCABULARY **Tell whether the angles or sides are *corresponding angles*, *corresponding sides*, or *neither*.**

1. $\angle C$ and $\angle L$

2. $\overline{AC}$ and $\overline{JK}$

3. $\overline{BC}$ and $\overline{KL}$

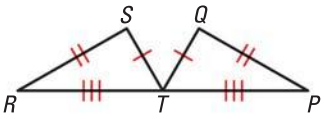
4. $\angle B$ and $\angle L$



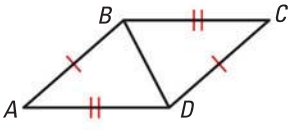
EXAMPLE 1 on p. 234 for Exs. 5–7

DETERMINING CONGRUENCE **Decide whether the congruence statement is true. *Explain* your reasoning.**

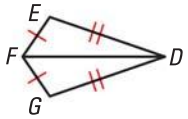
5. $\triangle RST \cong \triangle TQP$



6. $\triangle ABD \cong \triangle CDB$

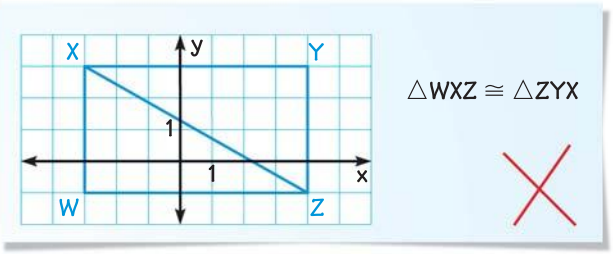


7. $\triangle DEF \cong \triangle DGF$



EXAMPLE 2
on p. 235
for Exs. 8–12

8. ERROR ANALYSIS *Describe* and correct the error in writing a congruence statement for the triangles in the coordinate plane.



ALGEBRA **Use the given coordinates to determine if $\triangle ABC \cong \triangle DEF$.**

9. $A(-2, -2)$, $B(4, -2)$, $C(4, 6)$, $D(5, 7)$, $E(5, 1)$, $F(13, 1)$

10. $A(-2, 1)$, $B(3, -3)$, $C(7, 5)$, $D(3, 6)$, $E(8, 2)$, $F(10, 11)$

11. $A(0, 0)$, $B(6, 5)$, $C(9, 0)$, $D(0, -1)$, $E(6, -6)$, $F(9, -1)$

12. $A(-5, 7)$, $B(-5, 2)$, $C(0, 2)$, $D(0, 6)$, $E(0, 1)$, $F(4, 1)$

EXAMPLE 3
on p. 236
for Exs. 13–15

USING DIAGRAMS **Decide whether the figure is stable. *Explain*.**

13.


14.


15.

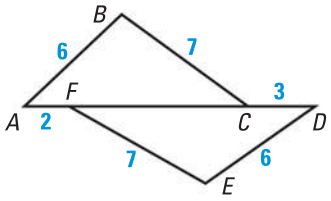

16. ★ MULTIPLE CHOICE Let $\triangle FGH$ be an equilateral triangle with point J as the midpoint of $\overline{FG}$. Which of the statements below is *not* true?

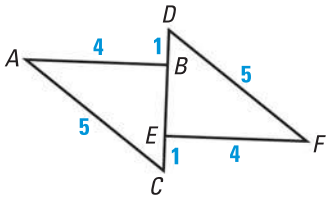
Ⓐ $\overline{FH} \cong \overline{GH}$ Ⓑ $\overline{FJ} \cong \overline{FH}$ Ⓒ $\overline{FJ} \cong \overline{GJ}$ Ⓓ $\triangle FHJ \cong \triangle GHJ$

17. ★ MULTIPLE CHOICE Let $ABCD$ be a rectangle separated into two triangles by $\overline{DB}$. Which of the statements below is *not* true?

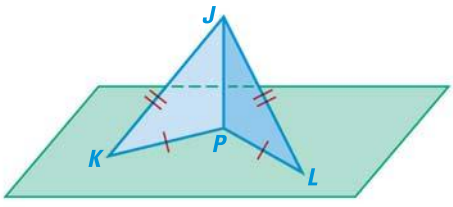
Ⓐ $\overline{AD} \cong \overline{CB}$ Ⓑ $\overline{AB} \cong \overline{AD}$ Ⓒ $\overline{AB} \cong \overline{CD}$ Ⓓ $\triangle DAB \cong \triangle BCD$

APPLYING SEGMENT ADDITION **Determine whether $\triangle ABC \cong \triangle DEF$. If they are congruent, write a congruence statement. *Explain* your reasoning.**

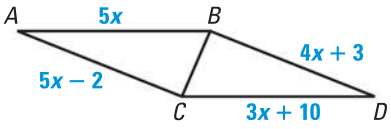
18.


19.


20. 3-D FIGURES In the diagram, $\overline{PK} \cong \overline{PL}$ and $\overline{JK} \cong \overline{JL}$. Show that $\triangle JPK \cong \triangle JPL$.



21. CHALLENGE Find all values of x that make the triangles congruent. *Explain*.



PROBLEM SOLVING

EXAMPLE 1 on p. 234 for Ex. 22

22. TILE FLOORS You notice two triangles in the tile floor of a hotel lobby. You want to determine if the triangles are congruent, but you only have a piece of string. Can you determine if the triangles are congruent? *Explain*.

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EXAMPLE 3 on p. 236 for Ex. 23

23. GATES Which gate is stable? *Explain* your reasoning.

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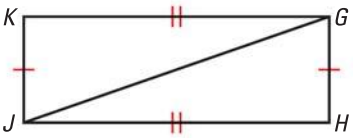
PROOF **Write a proof.**

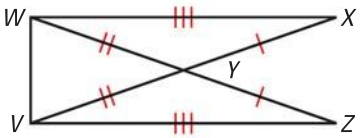
24. **GIVEN** ▶ $\overline{GH} \cong \overline{JK}$, $\overline{HJ} \cong \overline{KG}$

PROVE ▶ $\triangle GHJ \cong \triangle JKG$



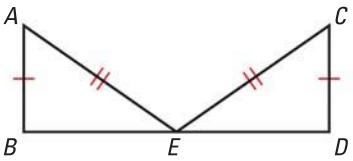
25. **GIVEN** ▶ $\overline{WX} \cong \overline{VZ}$, $\overline{WY} \cong \overline{VY}$, $\overline{YZ} \cong \overline{YX}$

PROVE ▶ $\triangle VWX \cong \triangle WVZ$



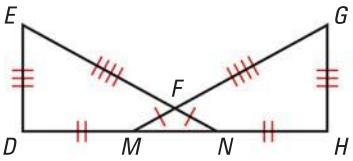
26. **GIVEN** ▶ $\overline{AE} \cong \overline{CE}$, $\overline{AB} \cong \overline{CD}$, E is the midpoint of $\overline{BD}$.

PROVE ▶ $\triangle EAB \cong \triangle ECD$



27. **GIVEN** ▶ $\overline{FM} \cong \overline{FN}$, $\overline{DM} \cong \overline{HN}$, $\overline{EF} \cong \overline{GF}$, $\overline{DE} \cong \overline{HG}$

PROVE ▶ $\triangle DEN \cong \triangle HGM$



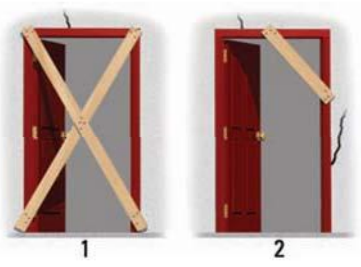
28. ★ **EXTENDED RESPONSE** When rescuers enter a partially collapsed building they often have to reinforce damaged doors for safety.

a. Diagonal braces are added to Door 1 as shown below. *Explain* why the door is more stable with the braces.

b. Would these braces be a good choice for rescuers needing to enter and exit the building through this doorway?

c. In the diagram, Door 2 has only a corner brace. Does this solve the problem from part (b)?

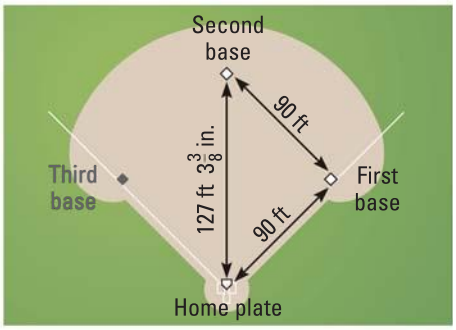
d. *Explain* why the corner brace makes the door more stable.



○ = WORKED-OUT SOLUTIONS on p. WS1

★ = STANDARDIZED TEST PRACTICE

29. BASEBALL FIELD To create a baseball field, start by placing home plate. Then, place second base 127 feet $3\frac{3}{8}$ inches from home plate. Then, you can find first base using two tape measures. Stretch one from second base toward first base and the other from home plate toward first base. The point where the two tape measures cross at the 90 foot mark is first base. You can find third base in a similar manner. *Explain* how and why this process will always work.



30. CHALLENGE Draw and label the figure described below. Then, identify what is given and write a two-column proof.

In an isosceles triangle, if a segment is added from the vertex between the congruent sides to the midpoint of the third side, then two congruent triangles are formed.

MIXED REVIEW

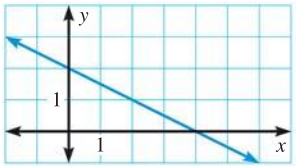
PREVIEW
Prepare for Lesson 4.4 in Exs. 31–33.

Find the slope of the line that passes through the points. *(p. 171)*

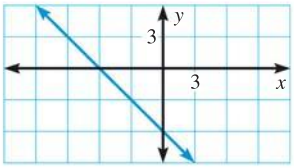
31. $A(3, 0), B(7, 4)$ **32.** $F(1, 8), G(-9, 2)$ **33.** $M(-4, -10), N(6, 2)$

Use the *x*- and *y*-intercepts to write an equation of the line. *(p. 180)*

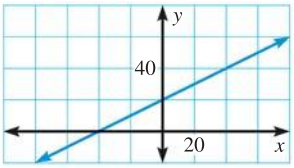
34.



35.



36.



37. Write an equation of a line that passes through $(-3, -1)$ and is parallel to $y = 3x + 2$. *(p. 180)*

QUIZ *for Lessons 4.1–4.3*

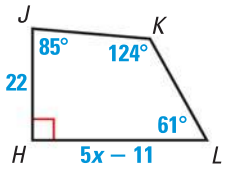
A triangle has the given vertices. Graph the triangle and classify it by its sides. Then determine if it is a right triangle. *(p. 217)*

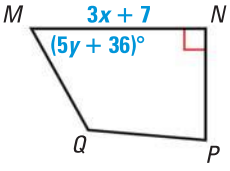
1. $A(-3, 0), B(0, 4), C(3, 0)$ **2.** $A(2, -4), B(5, -1), C(2, -1)$ **3.** $A(-7, 0), B(1, 6), C(-3, 4)$

In the diagram, *HJKL* ≅ *NPQM*. *(p. 225)*

4. Find the value of x.

5. Find the value of y.





6. Write a proof. *(p. 234)*

GIVEN ▶ $\overline{AB} \cong \overline{AC}$, $\overline{AD}$ bisects $\overline{BC}$.

PROVE ▶ $\triangle ABD \cong \triangle ACD$